Week 6
MATH 34B
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Given that there's a midterm this coming Friday based on the material from homeworks 3+4, I decided to compile some of the harder and less done questions from these homeworks. In this packet, there are two question from homework 1 and three from homework 2.
Here's how today's going to work. As usual, I will devote the first thirty minutes to letting you guys have a crack at these questions. Afterwards, I will present some of these problems. The main difference between today's section and the other sections we've had is that I will not be coming around to help, and there will be no collaboration. The intent of this is to simulate the conditions of an exam (where there will be neither collaboration allowed nor TAs walking you through problems). One thing I want to say is that, **I DO NOT CLAIM THIS WILL BE SIMILAR IN ANY WAY, SHAPE, OR FORM, TO THE ACTUAL MIDTERM.** In particular, I do not have any part in writing the midterm, nor did the professor have any part in the creation/compilation of this worksheet.

3.15 A smokestack deposits soot on the ground with a concentration inversely proportional to the square of the distance from the stack. With two smokestacks d miles apart, the concentration of the combined deposits on the line joining them, at a distance x from one stack, is given by $S = \frac{c}{x^2} + \frac{k}{(d-x)^2}$ where c and k are positive constants which depend on the quantity of smoke each stack is emitting. If $k = 4c$, find the point on the line joining the stacks where the concentration of the deposit is a minimum.

We want to minimize $S = \frac{c}{x^2} + \frac{k}{(d-x)^2}$

$$= \frac{c}{x^2} + \frac{4c}{(d-x)^2} = cx^{-2} + 4c(d-x)^{-2}$$

Taking derivative, we get

$$(-2)cx^{-3} + 4c(-2)(d-x)^{-3} \cdot (-1) = 0$$

set $=0$ to minimize.

$$\Rightarrow -2cx^{-3} = -8c(d-x)^{-3}$$

$$\Rightarrow x^{-3} = 4(d-x)^{-3}.$$

$(d-x)^3 = 4x^3$ (multiply both sides

cube root both sides... (i.e. take power to $1/3$).

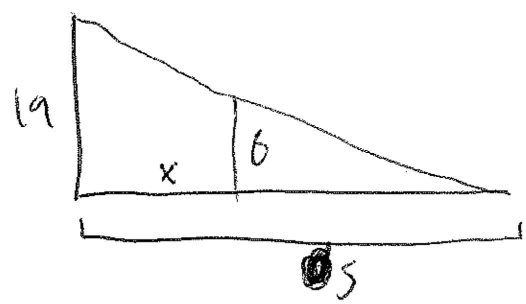
$$d - x = 4^{1/3} x$$

$$\Rightarrow x = \frac{d}{1+4^{1/3}}$$

3.20 A street light is at the top of a 19 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 4 ft/sec along a straight path. How fast is the tip of her shadow moving along the ground when she is 35 ft from the base of the pole?

(This is a related rates problem, so if you thought you could just forget about this, think again!)

First, let us draw a picture. We have

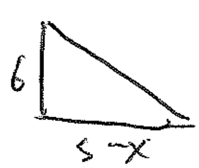


where x is distance ~~...~~ between woman and pole, s is distance between tip of shadow and pole.

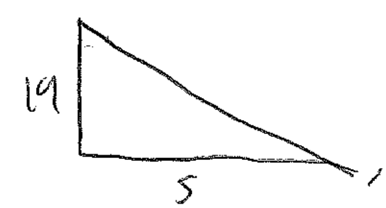
$\boxed{\text{NOTE: IN \underline{GENERAL}, X IS NOT 35!!!}}$

How fast tip of shadow, which is what we're looking for is exactly $\frac{ds}{dt}$. We know $\frac{dx}{dt} = 4$. If only we can relate the two...

Since



is similar to



we have relation $\frac{s}{19} = \frac{s-x}{6} \Rightarrow \frac{s}{19} = \frac{s}{6} - \frac{x}{6} \Rightarrow \frac{x}{6} = s\left(\frac{1}{6} - \frac{1}{19}\right)$

Now, taking $\frac{d}{dt}$ to both sides gives us $\frac{dx}{dt} \cdot \frac{1}{6} = \frac{ds}{dt}\left(\frac{1}{6} - \frac{1}{19}\right)$.

Since $\frac{dx}{dt} = 4$, we have $4 \cdot \frac{1}{6} = \frac{ds}{dt}\left(\frac{1}{6} - \frac{1}{19}\right)$

$\Rightarrow \frac{ds}{dt} = \left(\frac{4}{6}\right)\frac{1}{(1/6 - 1/19)}$ //

4.14 The speed of car A after t minutes is $8t\ m/s$.

How long will it take the car to travel 100/6 meters?

We have $v(t) = 8t$.

Normally, we would have $p(t) = \frac{8t^2}{2} + C = 4t^2 + C$.

However, at $t = 0$, we haven't travelled anywhere,

so $p(0) = 0$, so $C = 0 \Rightarrow p(t) = 4t^2$.

Now, set $p(t) = 100/6$

$$4t^2 = 100/6$$

$$t^2 = 100/24$$

$$t = \sqrt{\frac{100}{24}}.$$

4.16 How quickly a leaf grows is proportional how big [ie the surface area] the leaf is. If the area of the leaf grows from $2cm^2$ to $3cm^2$ in 3 days, how long will it take for the leaf's area to increase to 5 cm^2?

We have $\frac{dA}{dt} \sim A$, so $\frac{dA}{dt} = kA$

$\frac{dA}{A} = k\,dt$ ("divide by A and "multiply" by dt).

$\int \frac{dA}{A} = \int k\,dt$ ("integrate")

$\ln A = kt + C.$

(take both left and right to e). $e^{\ln A} = A = e^{kt+C} = e^C e^{kt}$

$A = Ce^{kt}$ (e^C a constant, so might as well call it C).

$2 = Ce^{k \cdot 0} = C$ (at day 0, it is $2cm^2$).

$\Rightarrow A = 2e^{kt}.$

$3 = 2e^{3 \cdot k}$ (at day 3, it is $3cm^2$).

$3/2 = e^{3k}$

$\ln(3/2) = 3k$

$k = 1/3 \ln(3/2).$

So, $A = 2e^{kt}$, where k is

how long for $5cm^2$? set $A = 5$

$\Rightarrow 5 = 2e^{kt}$

$\frac{5}{2} = e^{kt} \Rightarrow \ln(\frac{5}{2}) = kt$

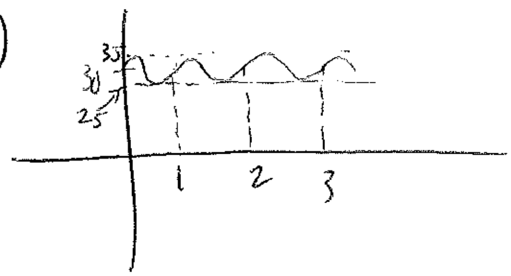
$\Rightarrow \frac{1}{k}\ln(\frac{5}{2}) = t$ //

4.27 An artery has a circular cross section of radius 4 millimeters. The speed at which blood flows along the artery fluctuates as the heart beats. The speed after t seconds is $30 + 5\sin(2\pi t)$ meters per second.

(a) Sketch on graph paper a graph of the speed over a 3 second time span.

(b) What volume of blood passes along the artery in one second?

a)



b) Since the cross section of artery is circle of radius 4 mm, we can think of all the blood flowing through as a cylinder, of height equal to how far a blood particle travels in one second.

In one second, blood travels $\int_0^1 30 + 5\sin(2\pi t)\,dt$

$$= 30t - 5\cos(2\pi t)\frac{1}{2\pi}\Big|_0^1$$

$$= 30 - \frac{5}{2\pi}\cos(2\pi(1)) - \left(-\frac{5}{2\pi}\cos(0)\right)$$

$$= 30 \text{ meters} = 30{,}000 \text{ mm}.$$

$$V = \pi r^2 h = \pi(4)^2(30{,}000)$$